JET FLOW AROUND A PLATE NEAR A WALL

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ABSTRACT: Chaplygin's method [1] has been extended by Fal'kovich [2] to the case of several characteristic velocities; it has here been used to solve the two-dimensional unsymmetrical problem of subsonic gas jet flow around a plate near a solid wall. The Zhukovskii-Roshko scheme [3, 4] has been used with a stagnant zone ahead of the plate. Formulas are derived for the current function, normal-pressure coefficient, and geometrical elements of the flow. The result is extended to the case of an incompressible fluid by passing to the limit.

1. A subsonic gas flow moves along the solid horizontal wall FME with a speed v_1 and strikes a plate AB of length l that lies at a distance h from the wall. The plate forms an angle $\alpha = \sigma\pi$ $(0 < \sigma < 0.5)$ with the negative direction of v_1 and divides the flow into two unsymmetrical branches. The upper branch is unbounded in width and bears an infinite amount of fluid, while the lower one has width δ and flow rate Q. The liquid near the plate is decelerated, the minimum velocity $v_0 < v_1$ on FME occurring at M. The velocity is $v_2 > v_1$ on the surfaces AC and BD. We place the origin at A, with the x-axis directed along v_1 (Fig. 1).

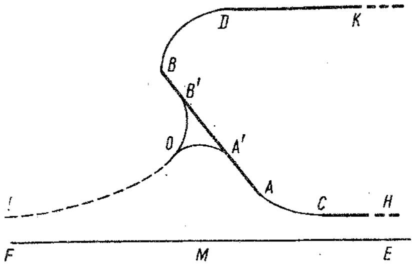

Fig. 1

The problem is solved via the Zhukovskii-Roshko scheme [3, 4] with a stagnant zone ahead of the plate. Chaplygin [5] considered jet flow around a plate with such a zone, with an arbitrary velocity on the jet surfaces bounding the stagnant zone (this velocity is to be found from additional conditions). Here it is assumed that this velocity equals v_0, the least velocity on the wall.

The jet theory gives [4] a low value for the normal-pressure coefficient, so we need a larger pressure P_0 in the stagnant zone in order to obtain a result close to the actual one, i. e., a lower v_0 on surfaces OA' and OB'. Since the flow surface can be taken as a rigid wall, we consider the subsonic flow of the gas in a channel in which one wall FME is straight and the other LOA'ACH is curved. The velocity drop in this channel will be the greater the larger the expansion of the channel near the plate (streamline LO is perpendicular to the plate at the branch point O) and the less the width of the channel to the left at infinity, and this is possible for h small and α small, since then LO undergoes the greatest change in slope. This problem (α and h small) is of the greatest interest, although the solution is mathematically applicable for any α and h allowed by the flow scheme.

We assume that the current function $\psi = 0$ on the branching lines OA'ACH and OB'BDK, while $\psi = -Q$ on FME.

A semicircle with a slot at $\theta = 0$ and radii τ_0 and τ_2 (Fig. 2) represents the entire region filled by the flow in the plane of the velocity hodograph with the polar coordinates θ and $\tau = v^2/v_{max}^2$. The following are the values that ψ must take along the boundaries of the hodograph region:

$$\begin{aligned}
&\psi = 0 \quad &&\text{for } \theta = -\sigma\pi, \quad &&\tau_0 \leqslant \tau \leqslant \tau_2 \\
&\psi = 0 \quad &&\text{for } \theta = \gamma\pi, \quad &&\tau_0 \leqslant \tau \leqslant \tau_2, \ \gamma = 1 - \sigma \\
&\psi = -Q \quad &&\text{for } \theta = 0, \quad &&\tau_0 \leqslant \tau \leqslant \tau_1 \\
&\psi = 0 \quad &&\text{for } \theta = 0, \quad &&\tau_1 \leqslant \tau \leqslant \tau_2
\end{aligned} \qquad (1.1)$$

$$\begin{aligned}
&\psi = 0 \quad &&\text{for } \tau = \tau_0, \quad &&-\sigma\pi \leqslant \theta \leqslant 0 \\
&\psi = 0 \quad &&\text{for } \tau = \tau_0, \quad &&0 \leqslant \theta \leqslant \gamma\pi \\
&\psi = 0 \quad &&\text{for } \tau = \tau_2, \quad &&-\sigma\pi \leqslant \theta \leqslant 0 \\
&\psi = 0 \quad &&\text{for } \tau = \tau_2, \quad &&0 \leqslant \theta \leqslant \gamma\pi
\end{aligned} \qquad (1.2)$$

The problem then reduces to an internal Dirichlet problem for Chaplygin's equation:

$$4\tau^2(1-\tau)\frac{\partial^2\psi}{\partial\tau^2} + 4\tau[1 + (\beta - 1)\tau]\frac{\partial\psi}{\partial\tau} + [1 - (2\beta + 1)\tau]\frac{\partial^2\psi}{\partial\theta^2} = 0$$

$$(\beta = 1/(\varkappa - 1), \ \varkappa = c_p/c_v) \qquad (1.3)$$

in the regions of the semicircle. As $\tau < 1/(2\beta + 1)$, Eq. (1.3) is of elliptic type in this region, and we seek a solution in the form

$$\psi_1(\tau, \theta) = Q\frac{\theta - \gamma\pi}{\gamma\pi} + \sum_{n=1}^{\infty}[A_n z_\lambda(\tau) + B_n \zeta_\lambda(\tau)]\sin\frac{n\theta}{\gamma},$$

$$\left(\lambda = \frac{n}{\gamma}\right),$$

$$\psi_2(\tau, \theta) = -Q\frac{\theta + \sigma\pi}{\sigma\pi} + \sum_{n=1}^{\infty}[(C_n z_\omega(\tau) + D_n \zeta_\omega(\tau)]\sin\frac{n\theta}{\sigma},$$

$$\left(\omega = \frac{n}{\sigma}\right),$$

$$\psi_3(\tau, \theta) = \sum_{n=1}^{\infty}[E_n z_\lambda(\tau) + F_n \zeta_\lambda(\tau)]\sin\frac{n\theta}{\gamma},$$

$$\psi_4(\tau, \theta) = \sum_{n=1}^{\infty}[G_n z_\omega(\tau) + H_n \zeta_\omega(\tau)]\sin\frac{n\theta}{\sigma}. \qquad (1.4)$$

Here the subscript to ψ is the number of the region in the semicircle (Fig. 2), while $z_\nu(\tau)$ is an integral of the equation

$$4\tau^2(1 - \tau)z_\nu'' + 4\tau[1 + (\beta - 1)\tau]z_\nu' - \nu^2[1 - (2\beta + 1)\tau] \ z_\nu = 0, \quad (\nu = \omega, \ \lambda), \qquad (1.5)$$

which is regular at $\tau = 0$ and which has been considered by Chaplygin [1], while $\zeta_\nu(\tau)$ is a second linearly independent integral of (1.5) derived by Lighthill [6] and Cherry [7, 8], which becomes the logarithmic solution of [7] when ν is a positive integer. Fal'kovich [2] was the first to use this integral in the theory of gas jets. The following is the Wronskian for these integrals:

$$W_\nu(\tau) = z_\nu'(\tau)\zeta_\nu(\tau) - \zeta_\nu'(\tau)z_\nu(\tau) = \nu(1 - \tau)^\beta\tau^{-1}, \qquad (1.6)$$

$$(\nu = \omega, \ \lambda),$$

where the coefficients A_n, B_n, ..., H_n have to be determined.

The ψ defined by (1.4) satisfy the boundary conditions of (1.1). We now specify that the boundary conditions of (1.2) are obeyed and also that the solutions meet the conditions for analytic continuation through the boundaries of the subregions, i. e.,

$$\psi_1 = \psi_3, \quad \frac{\partial\psi_1}{\partial\tau} = \frac{\partial\psi_3}{\partial\tau} \quad \text{for} \quad \tau = \tau_1, \ 0 \leqslant \theta \leqslant \gamma\pi \qquad (1.7)$$

$$\psi_2 = \psi_4, \quad \frac{\partial\psi_2}{\partial\tau} = \frac{\partial\psi_4}{\partial\tau} \quad \text{for} \quad \tau = \tau_1, \ -\sigma\pi \leqslant \theta \leqslant 0.$$

Conditions (1.2) and (1.7) give a system of 8n algebraic equations in 8n unknowns, which are solved for the coefficients to give the final solution in the form

$$\psi_1(\tau, \theta) = \frac{Q}{\gamma\pi}\left[\theta - \gamma\pi - 2\sum_{n=1}^{\infty} \chi_1(\tau)\frac{\sin\lambda\theta}{\lambda}\right],$$

$$\psi_2(\tau, \theta) = -\frac{Q}{\sigma\pi}\left[\theta + \sigma\pi - 2\sum_{n=1}^{\infty} \chi_2(\tau)\frac{\sin\omega\theta}{\omega}\right],$$

$$\psi_3(\tau, \theta) = \frac{2Q}{\gamma\pi}\sum_{n=1}^{\infty} \chi_3(\tau)\frac{\sin\lambda\theta}{\lambda},$$

$$\psi_4(\tau, \theta) = -\frac{2Q}{\sigma\pi}\sum_{n=1}^{\infty} \chi_4(\tau)\frac{\sin\omega\theta}{\omega}. \qquad (1.8)$$

Here

$$\chi_i(\tau) = \frac{T_\nu(\tau, \tau_2) - T_\nu(\tau, \tau_0)}{T_\nu(\tau_2, \tau_0)} - \frac{T_\nu'(\tau_1 \tau_2) - W_\nu(\tau_1)}{T_\nu(\tau_2, \tau_0)}\frac{T_\nu(\tau, \tau_0)}{W_\nu(\tau_1)},$$

$$\chi_j(\tau) = \frac{T_\nu(\tau_1, \tau_0) - W_\nu(\tau_1)}{T_\nu(\tau_2, \tau_0)}\frac{T_\nu(\tau, \tau_2)}{W_\nu(\tau_1)}$$

$$(i = 1, \quad j = 3, \quad \nu = \lambda; \quad i = 2, \quad j = 4, \quad \nu = \omega),$$

$$T_\nu(\tau_i, \tau_j) = z_\nu(\tau_i)\zeta_\nu(\tau_j) - z_\nu(\tau_j)\zeta_\nu(\tau_i), \quad T_\nu(\tau_i, \tau_i) = 0,$$

$$T_\nu'(\tau_i, \tau_j) = [T_\nu'(\tau, \tau_j)]_{\tau=\tau_i}, \quad T_\nu'(\tau_i, \tau_i) = W_\nu(\tau_i)$$

$$(i, j = 0, 1, 2; \ \nu = \omega, \lambda). \qquad (1.9)$$

We also note that

$$\chi_i'(\tau_0) = \frac{T_\nu'(\tau_0, \tau_2) - W_\nu(\tau_0)}{T_\nu(\tau_2, \tau_0)} - \frac{T_\nu'(\tau_1, \tau_2) - W_\nu(\tau_1)}{T_\nu(\tau_2, \tau_0)}\frac{W_\nu(\tau_0)}{W_\nu(\tau_1)},$$

$$\chi_j'(\tau_2) = \frac{T_\nu'(\tau_1, \tau_0) - W_\nu(\tau_1)}{T_\nu(\tau_2, \tau_0)}\frac{W_\nu(\tau_2)}{W_\nu(\tau_1)},$$

$$\chi_i(\tau_0) = -1, \quad \chi_j(\tau_2) = 0, \quad \chi_i(\tau_1) = \chi_j(\tau_1) = -1,$$

$$\chi_i'(\tau_1) + \chi_j'(\tau_1) = 0,$$

$$(i = 1, \ j = 3, \ \nu = \lambda; \ i = 2, \ j = 4, \ \nu = \omega). \qquad (1.10)$$

We introduce the following exactly zero binomial into the expression for $\chi_i(\tau)$ in order to unify the expressions for $\chi_i'(\tau_0)$ and $\chi_j'(\tau_2)$ together with the functions $H(\nu, \tau)$, $\Pi(\nu, \tau)$, and $\Omega(\nu, \tau)$, which are defined below by (2.4), (2.10), and (2.14):

$$\frac{W_\nu(\tau_1) T_\nu(\tau, \tau_0)}{T_\nu(\tau_2, \tau_0) W_\nu(\tau_1)} - \frac{T_\nu(\tau, \tau_0)}{T_\nu(\tau_2, \tau_0)}.$$

2. Consider the relation between the parameters of the problem. The following general formula applies along any jet surface:

$$dy = 2\tau\frac{(1-\tau)^{-\beta}}{v}\frac{\partial\psi}{\partial\tau}\sin\theta\, d\theta. \qquad (2.1)$$

We substitute into (2.1) the $\psi_4(\tau, \theta)$ of (1.8), integrate with respect to θ along AC ($\tau = \tau_2 = \text{const}$) from $\theta = \sigma\pi$ ($y = 0$) to $\theta = 0$ ($y = y_C$), use (1.6) and (1.10), and employ the expression for the flow rate

$$Q = \delta v_1(1 - \tau_1)^\beta, \qquad (2.2)$$

which gives us the ordinate of point C

$$y_C = -\delta H(\omega, \tau_1). \qquad (2.3)$$

Here

$$H(\nu, \tau) = 4\tau\left(\frac{1-\tau_1}{1-\tau}\right)^\beta\left(\frac{\tau_1}{\tau_2}\right)^{1/2}\frac{\sin k\pi}{k\pi}\times$$

$$\times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\nu^2-1}\frac{T_\nu'(\tau, \tau_0) - W_\nu(\tau)}{T_\nu(\tau_2, \tau_0)}$$

$$(\nu = \lambda, \ k = \gamma; \ \nu = \omega, \ k = \sigma). \qquad (2.4)$$

Similarly we find the ordinate of point D:

$$y_D = l\sin\sigma\pi + \delta H(\lambda, \tau_1). \qquad (2.5)$$

We add the $|y_C|$ of (2.3) to the y_D of (2.5) to get an expression for the width of the cavity:

$$a = l\sin\sigma\pi + \delta[H(\lambda, \tau_1) + H(\omega, \tau_1)]. \qquad (2.6)$$

Since $\delta + |y_C| = h$ (Fig. 1), we have

$$\delta = h[1 + H(\omega, \tau_1)]^{-1}. \qquad (2.7)$$

We turn the x- and y-axes counterclockwise through the angle

$$\theta_1 = {}^1/_2\,\pi - \sigma\pi = \gamma\pi - {}^1/_2\,\pi$$

and get a new coordinate system x'Ay' in which along any streamline

$$dy' = \frac{(1-\tau)^{-\beta}}{v}\times$$

$$\times\left[2\tau\frac{\partial\psi}{\partial\tau}\sin\theta'\,d\theta' - \frac{1-(2\beta+1)\tau}{2\tau(1-\tau)}\frac{\partial\psi}{\partial\theta'}\sin\theta'\,d\tau\right]. \qquad (2.8)$$

We have $\theta' = \pi/2$ and $\theta' = -\pi/2$ along parts B'B and AA' of the plate, respectively ($\theta' = \theta - \theta_1$), and (2.8) can be written as

$$dy' = \pm\frac{1-(2\beta+1)\tau}{2v_{max}\tau^{3/2}(1-\tau)^{\beta+1}}\left(\frac{\partial\psi}{\partial\theta'}\right)_{\theta'=\mp{}^1/_2\pi} d\tau, \qquad (2.9)$$

where + corresponds to AA' and − to B'B. We integrate (2.9) from τ_0 to τ_2 to find the length l_1 of part B'B:

$$l_1 = -\frac{1}{2v_{max}}\left[\int_{\tau_0}^{\tau_1}\frac{1-(2\beta+1)\tau}{\tau^{3/2}(1-\tau)^{\beta+1}}\left(\frac{\partial\psi_1}{\partial\theta'}\right)_{\theta'={}^1/_2\pi} d\tau + \right.$$

$$\left. + \int_{\tau_1}^{\tau_2}\frac{1-(2\beta+1)\tau}{\tau^{3/2}(1-\tau)^{\beta+1}}\left(\frac{\partial\psi_3}{\partial\theta'}\right)_{\theta'={}^1/_2\pi} d\tau\right].$$

We calculate the integrals with use of (1.8), (1.10), (2.4), and (2.7) together with the notation

$$\Pi(\nu, \tau) = 4\tau\left(\frac{1-\tau_1}{1-\tau}\right)^\beta\left(\frac{\tau_2}{\tau_0}\right)^{1/2}\frac{\sin k\pi}{k\pi}\times$$

$$\times\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\nu^2-1}\frac{T_\nu'(\tau, \tau_2) - W_\nu(\tau)}{T_\nu(\tau_2, \tau_0)},$$

$$\Delta = \left(\frac{1-\tau_1}{1-\tau_0}\right)^\beta\left(\frac{\tau_1}{\tau_0}\right)^{1/2} \qquad (\nu = \lambda, \ k = \gamma; \ \nu = \omega, \ k = \sigma), \qquad (2.10)$$

whereupon elementary steps give

$$l_1 = h[1 - \Delta + H(\lambda, \tau_1) + \Pi(\lambda, \tau_0)] -$$

$$-\Pi(\lambda, \tau_1)]\{[1 + H(\omega, \tau_1)]\sin\sigma\pi\}^{-1}. \qquad (2.11)$$

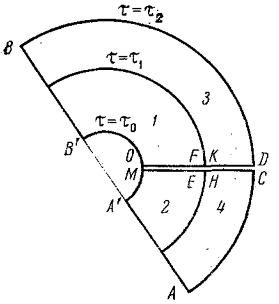

Fig. 2

In deriving (2.11) we have used the equalities [1]

$$\int \frac{1-(2\beta+1)\tau}{\tau^{3/2}(1-\tau)^{\beta+1}} \chi_i(\tau)\, d\tau = \frac{2}{\nu^2-1}\frac{(1-\tau)^{-\beta}}{\tau^{1/2}}[\chi_i(\tau)+2\tau\chi_i'(\tau)]$$

$$(i=1,3{:}\ \nu=\lambda;\ i=2,4;\ \nu=\omega),$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\nu^2-1} = \frac{1}{2}\left(1-\frac{k\pi}{\sin k\pi}\right)$$

$$(\nu=\lambda,\ k=\gamma;\ \nu=\omega,\ k=\sigma). \qquad (2.12)$$

We find the length l_2 of AA' similarly:

$$l_2 = h\,[\Delta - 1 + \mathrm{H}(\omega,\tau_1) + \Pi(\omega,\tau_0) - \Pi(\omega,\tau_1)] \times$$

$$\times \{[1+\mathrm{H}(\omega,\tau_1)]\sin\sigma\pi\}^{-1}. \qquad (2.13)$$

We substitute into (2.1) for $\psi_2(\tau,\theta')$ and then the $\psi_1(\tau,\theta')$ from (1.8), integrate with respect to θ along A'OB' ($\tau=\tau_0=$ const) from $\theta'=-\pi/2$ ($y'=y_A'$) to $\theta'=\pi/2$ ($y'=y_B'$), and use (1.10), (2.2), (2.4), and (2.10) together with the notation

$$\Omega(\nu,\tau) = 4\tau\left(\frac{1-\tau_1}{1-\tau}\right)^{\beta}\left(\frac{\tau_1}{\tau_0}\right)^{1/2}\frac{\sin k\pi}{k\pi}\times$$

$$\times\sum_{n=1}^{\infty}\frac{t_\nu}{\nu^2-1}\,\frac{T_\nu'(\tau,\tau_2)-W_\nu(\tau)}{T_\nu(\tau_2,\tau_0)},$$

$$t_\nu = -(1)^{n-1}+\cos k\pi$$

$$(\nu=\lambda,\ k=\gamma;\ \nu=\omega,\ k=\sigma) \qquad (2.14)$$

to get the length l_3 of A'B' in the form

$$l_3 = h\,[\Omega(\lambda,\tau_0) - \Omega(\lambda,\tau_1) + \Omega(\omega,\tau_0) - \Omega(\omega,\tau_1)] \times \qquad (2.15)$$

$$\times \{[1+\mathrm{H}(\omega,\tau_1)]\sin\sigma\pi\}^{-1}.$$

Addition of (2.11), (2.13), and (2.15) gives

$$l = h\,[\Pi(\lambda,\tau_0) - \Pi(\lambda,\tau_1) + \Pi(\omega,\tau_0) -$$

$$-\Pi(\omega,\tau_1) + \Omega(\lambda,\tau_0) - \Omega(\lambda,\tau_1) +$$

$$+\Omega(\omega,\tau_0) - \Omega(\omega,\tau_1) + \mathrm{H}(\lambda,\tau_1) + \mathrm{H}(\omega,\tau_1)]\times$$

$$\times\{[1+\mathrm{H}(\omega,\tau_1)]\sin\sigma\pi\}^{-1}. \qquad (2.16)$$

We use (2.7) and (2.16) to transform (2.6) to

$$a = h\,\{\Pi(\lambda,\tau_0) - \Pi(\lambda,\tau_1) + \Pi(\omega,\tau_0) -$$

$$-\Pi(\omega,\tau_1) + \Omega(\lambda,\tau_0) - \Omega(\lambda,\tau_1) +$$

$$+\Omega(\omega,\tau_0) - \Omega(\omega,\tau_1) + 2\,[\mathrm{H}(\lambda,\tau_1) + \mathrm{H}(\omega,\tau_1)]\}\times$$

$$\times[1+\mathrm{H}(\omega,\tau_1)]^{-1}. \qquad (2.17)$$

Relations (2.2), (2.7), (2.16), and (2.17) define Q, δ, the speed at the jet surfaces of zone τ_0, and also a as functions of τ_1, τ_2, h, and l.

3. The resultant pressure force R on AB is determined by integration along the plate, which gives

$$R = I_1 + I_2 + p_0 l_3 - p_2 l,$$

$$I_1 = \int_{B'}^{B} p\,dy', \qquad I_2 = \int_{A'}^{A} p\,dy'. \qquad (3.1)$$

Here p is the pressure ahead of the plate, p_0 is the pressure in the stagnant zone, and p_2 is the pressure behind the plate. We use (2.9) with

$$p = p^0(1-\tau)^{\beta+1},\quad p_i = p^0(1,-\tau_i)^{\beta+1},$$

$$p^0 = v_{\max}^2/2(\beta+1) \quad (i=0,\ 1,\ 2), \qquad (3.2)$$

in which p_0 is the gas pressure at the branch point to put I_1 as

$$I_1 = -\frac{p^0}{2v_{\max}}\left[\int_{\tau_0}^{\tau_1}\frac{1-(2\beta+1)\tau}{\tau^{3/2}}\left(\frac{\partial\psi_1}{\partial\theta'}\right)_{\theta'=1/2\pi} d\tau + \right.$$

$$\left. + \int_{\tau_1}^{\tau_2}\frac{1-(2\beta+1)\tau}{\tau^{3/2}}\left(\frac{\partial\psi_3}{\partial\theta'}\right)_{\theta'=1/2\pi} d\tau\right]. \qquad (3.3)$$

The following formula [1] is used to calculate the integrals in conjunction with (1.8):

$$\int \tau^{-3/2}[1-(2\beta+1)\tau]\,\chi_i(\tau)\,d\tau =$$

$$= \frac{2}{\nu^2-1}\left\{\frac{1-\tau}{\sqrt{\tau}}[\chi_i(\tau)+2\tau\chi_i'(\tau)] + 2(\beta+1)\tau^{1/2}\chi_i(\tau)\right\},$$

and we use (1.10), (2.2), (2.4), (2.7), (2.10), and (2.12) to transform (3.3) to

$$I_1 = p^0 h\,\langle(1-\tau_1)^{\beta}\,\tau_1^{1/2}\times$$

$$\times\{[1+(2\beta+1)\tau_1]\tau_1^{-1/2} - [1+(2\beta+1)\tau_0]\,\tau_0^{-1/2}\} +$$

$$+\mathrm{H}(\lambda,\tau_1)(1-\tau_2)^{\beta+1} + (1-\tau_0)^{\beta+1}[\Pi(\lambda,\tau_0) - \Pi(\lambda,\tau_1)]\rangle\times$$

$$\times\{[1+\mathrm{H}(\omega,\tau_1)]\sin\pi\}^{-1}. \qquad (3.4)$$

An analogous expression may be derived for I_2:

$$I_2 = p^0 h\,\langle(1-\tau_1)^{\beta}\,\tau_1^{1/2}\{[1+(2\beta+1)\tau_0]\,\tau_0^{1/2} -$$

$$-[1+(2\beta+1)\tau_1]\tau_1^{-1/2}\} + \mathrm{H}(\omega,\tau_1)(1-\tau_2)^{\beta+1} +$$

$$+(1-\tau_0)^{\beta+1}[\Pi(\omega,\tau_0) - \Pi(\omega,\tau_1)]\rangle\times$$

$$\times\{1+\mathrm{H}(\omega,\tau_1)]\sin\sigma\pi\}^{-1}. \qquad (3.5)$$

Substitution of (3.2), (3.4), (3.5), (2.15), and (2.16) into (3.1) gives

$$R = p^0 h[(1-\tau_0)^{\beta+1} - (1-\tau_2)^{\beta+1}]\times$$

$$\times[\Omega(\lambda,\tau_0) - \Omega(\lambda,\tau_1) + \Omega(\omega,\tau_0 - \Omega(\omega,\tau_1) + \Pi(\lambda,\tau_0) -$$

$$-\Pi(\lambda,\tau_1) + \Pi(\omega,\tau_0) - \Pi(\omega,\tau_1)]\times$$

$$\times\{[1+\mathrm{H}(\omega,\tau_1]\sin\sigma\pi\}^{-1}.$$

This readily gives the normal-pressure coefficient as

$$c_n = \varepsilon F(\tau_0,\tau_1,\tau_2)\,[\Omega(\lambda,\tau_0) - \Omega(\lambda,\tau_1) +$$

$$+\Omega(\omega,\tau_0) - \Omega(\omega,\tau_1) + \Pi(\lambda,\tau_0) - \Pi(\lambda,\tau_1) +$$

$$+\Pi(\omega,\tau_0) - \Pi(\omega,\tau_1)]\quad\{[1+\mathrm{H}(\omega,\tau_1)]\sin\sigma\pi\}^{-1},$$

$$\varepsilon = \frac{h}{l}\quad F(\tau_0,\tau_1,\tau_2) =$$

$$= \frac{1-\tau_0}{(\beta+1)\tau_1}\left(\frac{1-\tau_0}{1-\tau_1}\right)^{\beta}\left[1-\left(\frac{1-\tau_2}{1-\tau_0}\right)^{\beta+1}\right]. \qquad (3.6)$$

We note that

$$\lim F(\tau_0,\tau_1,\tau_2) = (v_2^2 - v_0^2)v_1^{-2} \text{ при } v_{\max}\to\infty. \qquad (3.7)$$

Equations (2.7), (2.16), (2.17), and (3.6) serve to solve the problem.

4. The result is readily extended to the case of an incompressible liquid. The expressions [1, 8] for $z_\nu(\tau)$ and $\zeta_\nu(\tau)$ are used with (1.6) and (1.9) to find the limits

$$\lim_{v_{\max}\to\infty} \tau_i\,\frac{T_\nu'(\tau_i,\tau_2) - W_\nu(\tau_i)}{T_\nu(\tau_2,\tau_0)} = \frac{\nu}{2}\,\frac{q_{0i}^{\ \nu}(1-q_{i2}^{\ \nu})^2}{1-q_{02}^{\ 2\nu}},$$

$$q_{ij} = \left(\frac{\tau_i}{\tau_j}\right)^{1/2} = \frac{v_i}{v_j},$$

$$\lim_{v_{\max}\to\infty} \tau_i\,\frac{T_\nu'(\tau_i,\tau_0) - W_\nu(\tau_i)}{T_\nu(\tau_2,\tau_0)} = \frac{\nu}{2}\,\frac{q_{i2}^{\ \nu}(1-q_{0i}^{\ \nu})^2}{1-q_{02}^{\ 2\nu}} \quad (\nu=\omega,\ \lambda)$$

$$(i,\ j=0,\ 1,\ 2). \qquad (4.1)$$

We pass to the limit for $v_{max} \to \infty$ $(\tau \to 0)$ in (2.7), (2.16), (2.17), and (3.6) in conjunction with (3.7) and (4.1) to get these expressions as

$$\delta = h\,[1 + H_0(\omega, v_1)]^{-1},$$

$$l = h[\Pi_0(\lambda, v_0) - \Pi_0(\lambda, v_1) + \Pi_0(\omega, v_0) -$$

$$- \Pi_0(\omega, v_1) + \Omega_0(\lambda, v_0) - \Omega_0(\lambda, v_1) + \Omega_0(\omega, v_0) -$$

$$- \Omega_0(\omega, v_1) + H_0(\lambda, v_1) + H_0(\omega, v_1)]\ \{[1 + H_0(\omega, v_1)] \sin \sigma\pi\}^{-1},$$

$$a = h\{\Pi_0(\lambda, v_0) - \Pi_0(\lambda, v_1) + \Pi_0(\omega, v_0) -$$

$$- \Pi_0(\omega, v_1) + \Omega_0(\lambda, v_0) - \Omega_0(\lambda, v_1) + \Omega_0(\omega, v_0) -$$

$$- \Omega_0(\omega, v_1) + 2[H_0(\lambda, v_1) + H_0(\omega, v_1)]\}\ [1 + H_0(w, v_1)]\}^{-1},$$

$$c_n = \varepsilon q_{12}^{-2}(1 - q_{02}^2)[\Pi_0(\lambda, v_0) -$$

$$- \Pi_0(\lambda, v_1) + \Pi_0(\omega, v_0) - \Pi_0(\omega, v_1) + \Omega_0(\lambda, v_0), - \Omega_0(\lambda, v_1) +$$

$$+ \Omega_0(\omega, v_0) - \Omega_0(\omega, v_1)]\ \{[1 + H_0(\omega, v_1)] \sin \sigma\pi\}^{-1}.$$

Here we have used the symbols

$$H_0(\nu, v_i) = \lim_{\tau_i \to 0} H(\nu, \tau_i) =$$

$$2q_{12} \frac{\sin \sigma\pi}{\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2 - k^2} \frac{q_{i2}^{\ \nu}(1 - q_{0i}^{\ \nu})^2}{1 - q_{02}^{\ 2\nu}},$$

$$\Pi_0(\nu, v_i) = \lim_{\tau_i \to 0} \Pi(\nu, \tau_i) =$$

$$2q_{10} \frac{\sin \sigma\pi}{\pi} \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2 - k^2} \frac{q_{0i}^{\ \nu}(1 - q_{i2}^{\ \nu})^2}{1 - q_{02}^{\ 2\nu}},$$

$$\Omega_0(\nu, v_i) = \lim_{\tau_i \to 0} \Omega(\nu, \tau_i) =$$

$$2q_{10} \frac{\sin \sigma\pi}{\pi} \sum_{n=1}^{\infty} t_\nu \frac{n}{n^2 - k^2} \frac{q_{0i}^{\ \nu}(1 - q_{i2}^{\ \nu})^2}{1 - q_{02}^{\ 2\nu}}$$

$$(\nu = \lambda,\ k = \gamma;\ \nu = \omega,\ k = \sigma;\ i = 0,1).$$

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